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ON THE STRUCTURE OF QUASITRANSVERSE ELASTIC SHOCK WAVES*

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The structure of quasitransverse shock waves in a slightly anisotropic medium in the presence of dissipation due to viscosity is investigated. The existence of a shock structure "responsible" for ambiguity of the solution of a selfsimilar problem about waves excited in a half-space is demonstrated. The question of the existence of a structure for the remaining quasitransverse shock waves is discussed.

It is shown in the analysis of gas dynamics /1/ and certain other /2-5/ problems that selection of the discontinuities that should be utilized to construct solutions should not be constrained, in many cases, by just conditions for a non-decrease in entropy and by evolutionarity conditions. Confirmation of the requirement often utilized for the existence of the structure of a discontinuity /1, 3, 4, 5/ is especially important and interesting in cases when ambiguity of the solution of problems occurs, as in /1-4/ since it assists in selecting the unique solution that can actually be realized.

Ambiguity of the solution for certain ranges of the problem parameters was detected /6, 7/ when constructing solutions of selfsimilar problems in a prestressed or generally weakly anisotropic elastic medium for given initial strains and strains different from the initial on the half-space boundary. Quasitransverse shock waves that satisfy the condition of a non-decrease in entropy and the evolutionarity conditions were utilized in constructing the solutions. It can be suspected that, as in /1-4/, the ambiguity of the solution of self-similar problems is a result of the fact that not all the shock waves mentioned possess a structure, i.e., a continuous solution of a certain more-complete system of equations taking account of the dissipative processes proceeding in a narrow zone corresponding to the discontinuity in the solution of the original equations.

We will make a general remark here concerning the further content of the research where (as in /1-4/) only a stationary shock wave structure is examined. In cases when there is no stationary structure and the appropriate discontinuity necessarily occurs in the solution of the problem, a non-stationary structure is apparently realized. A well-known example of this kind is the hydraulic jump whose structure is turbulent. In those cases when there are several solutions of the problems, preference should obviously be given to solutions containing discontinuities possessing a stationary structure (as is done in /1-5/). From this viewpoint, shocks encountered in the solution only in the case of ambiguity, when tere is a competing solution not containing a shock of this kind, are of greatest interest for investigating the stationary structure.

The structure of quasitransverse shocks for which the necessary conditions for existence (evolutionarity and non-decrease of the entropy) are satisfied /8, 9/, is investigated below. Terms taking account of the additional stresses caused by viscosity are added as a dissipative mechanism to the dynamical elasticity theory equations. This is the simplest of the dissipative mechanisms used in the theory of a solid deformable body and ensures continuity of the solutions. It can be hoped that if ambiguity of the solution of the selfsimilar problem is associated with the absence of a structure for part of the discontinuities, then this should already have been detected in this model.

1. We consider the motion in the form of plane waves parallel to a certain plane which we select as the coordinate plane x_1x_2 of a Lagrange coordinate system $x_1, x_2, x_3 = x$. The quantities x_1, x_2, x correspond to rectangular Cartesian coordinates in the non-deformed state of the medium. Initial deformation of the medium, if it exists, is considered homogeneous:

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$\varepsilon_{ij}^{\circ} = \text{const}$. The components $\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{12}$ in the wave under consideration do not change. We will describe the change in the deformed state in the wave by the functions $\partial w_i/\partial x = u_i(x, t)$, where w_i are displacement vector components. The system of equations of a viscoelastic medium can here by written in the form

$$\rho_0 \frac{\partial v_i}{\partial t} = f_{ij} \frac{\partial u_j}{\partial x} + \rho_0 \frac{\partial}{\partial x}\left(\nu \frac{\partial v_i}{\partial x}\right), \quad i, j = 1, 2, 3 \tag{1.1}$$
$$\frac{\partial u_i}{\partial t} = \frac{\partial v_i}{\partial x}, \quad f_{ij} = \frac{\partial^2 \Phi}{\partial u_i \partial u_j}$$

Here ρ_0 is the density of the medium in the undeformed state, $v_i = \partial w_i/\partial t$ are the velocity vector components, $\Phi = \rho_0 U(\varepsilon_{ij}, S)$ is the elastic potential, U and S are the internal energy and entropy per unit mass, τ_{ij} is Green's finite strain tensor, and ν is the coefficient of kinematic viscosity. Not written down here is the energy equation which enables us to find the change in the entropy S in the flow. For low amplitude waves this change is small and exerts no influence on the dynamic equations (1.1) /8-10/.

If non-linearity and anisotropy effects are small, by using the third and sixth equations of (1.1), it is possible to express u_3 and v_3 approximately in terms of u_1 and u_2 for a quasi-transverse wave and to reduce system (1.1) to the form /11/ (see /12/ also, where an analogous operation was carried out for $\nu = 0$ for simple waves in an isotropic body)

$$\rho_0 \frac{\partial v_\alpha}{\partial t} = (\mu\delta_{\alpha\beta} + h_{\alpha\beta}) \frac{\partial u_\beta}{\partial x} + \rho_0 \frac{\partial}{\partial x}\left(\nu \frac{\partial v_\alpha}{\partial x}\right) \tag{1.2}$$
$$\frac{\partial u_\alpha}{\partial t} = \frac{\partial v_\alpha}{\partial x}, \quad h_{\alpha\beta} = \frac{\partial^2 F}{\partial u_\alpha \partial u_\beta}, \quad \alpha, \beta = 1, 2$$

Here $F = F(u_1, u_2)$ is a function expressed in terms of $\Phi(u_1, u_2, u_3)$. If the anisotropy of the initial state of the medium is due just to initial strains while the medium is isotropic in the undeformed state and its elastic potential can be taken in the form of the expansion /12/

$$\Phi = {}^1/_2\lambda I_1^2 + \mu I_2 + \beta I_1 I_2 + \gamma I_3 + \delta I_1^3 + \eta I_2^2 + \rho_0 T_0 (S - S_0) \tag{1.3}$$
$$I_1 = \varepsilon_{ii}, \ I_2 = \varepsilon_{ij}\varepsilon_{ij}, \ I_3 = \varepsilon_{ij}\varepsilon_{jk}\varepsilon_{ki}$$

the function $F(u_1, u_2)$ has the form /10/

$$F(u_1, u_2) = {}^1/_2 (g_{11}^{\circ} u_1^2 + g_{22}^{\circ} u_2^2) - {}^1/_8 \varkappa (u_1^2 + u_2^2)^2 \tag{1.4}$$
$$\varkappa = \mu + (\mu + \beta + {}^3/_2\gamma)^2/(\lambda + \mu) - 2\eta$$
$$g_{11}^{\circ} = \alpha - \mu - g, \ g_{22}^{\circ} = \alpha - \mu + g, \ g = (2\mu - {}^3/_2\gamma)(\varepsilon_{22} - \varepsilon_{11})$$
$$\alpha = \mu + 2bI_1^{\circ} - (\mu + {}^3/_4\gamma)(\varepsilon_{11} + \varepsilon_{22}), \ 2b = \lambda + 2\mu + \beta + {}^3/_2\gamma$$

As follows from (1.4), the quantity g is the single parameter which introduces the anisotropy caused by preliminary strain into the solution. As is shown in /10/, in the case of small anisotropy of a general kind, the function F will be determined as before by the equality (1.4) but with other g_{11}° and g_{22}°.

2. Let us investigate the structure of the quasitransverse discontinuities. Let W be the velocity of the discontinuity. We will seek solutions of system (1.2) of the form $u_\alpha = u_\alpha(\xi)$, $v_\alpha = v_\alpha(\xi)$, $\xi = -x + Wt$, $\alpha = 1, 2$ such that u_α will tend to constant values as $\xi \to \pm\infty$. As $\nu \to 0$ these solutions evidently go over into discontinuities in which the change in the quantities agrees with the change in the quantities in the corresponding quasitransverse shock /10/. The functions $u_\alpha(\xi)$ and $v_\alpha(\xi)$ should satisfy the equations

$$\rho_0 W \frac{dv_\alpha}{d\xi} = -(\mu\delta_{\alpha\beta} + h_{\alpha\beta}) \frac{du_\beta}{d\xi} + \nu\rho_0 \frac{d^2 v_\alpha}{d\xi^2}$$
$$-\frac{dv_\alpha}{d\xi} = W \frac{du_\alpha}{d\xi}, \quad \alpha, \beta = 1, 2$$

The function v_α can be eliminated from the system while the equations are integrated once with respect to ξ. Using the fact that $h_{\alpha\beta}$ are the second derivatives of F the equations can be reduced to the form

$$\rho_0 \nu W \frac{du_\alpha}{d\xi} = \frac{\partial q}{\partial u_\alpha}, \quad \alpha = 1, 2 \tag{2.1}$$
$$q = {}^1/_2 (\lambda_0 W^2 - \mu)(u_1^2 + u_2^2) - F + A_1 u_1 + A_2 u_2$$

The constants of integration A_α are the momentum flux in the direction of the axes x_α and can be determined from the conditions for $\xi = -\infty$, i.e., in front of the shock $u_1 = U_1$, $u_2 = U_2$, $(du_\alpha/d\xi)_\infty = 0$. Note that (2.1) denotes that the integral curves are orthogonal to the lines $q = \text{const}$. By virtue of (2.1)

$$\frac{dq}{d\xi} = \frac{\partial q}{\partial u_\alpha}\frac{du_\alpha}{d\xi} = (\rho_0 \nu W)^{-1}\left[\left(\frac{du_1}{d\xi}\right)^2 + \left(\frac{du_2}{d\xi}\right)^2\right] > 0$$

We note that the viscosity was taken as a scalar in writing (1.2). For anisotropic viscosity the viscous term in this and subsequent equations will change form, where $\nu\partial u_\alpha/\partial x$ and $\nu du_\alpha/d\xi$ in (2.1) is replaced by $\nu_{\alpha\beta}\partial u_\beta/\partial x$ and $\nu_{\alpha\beta}du_\beta/d\xi$ with positive-definite matrices $\nu_{\alpha\beta}$. For an aribtrary matrix $\nu_{\alpha\beta}$ the integral curves of the new Eqs.(2.1) can make an arbitrary angle with the lines $q = \text{const}$ but the inequality $dq/d\xi > 0$ is conserved.

Using the explicit form of the function $F(u_1, u_2)$ we obtain

$$\begin{aligned}
\rho_0 \nu W \frac{du_1}{d\xi} &= \left[\rho_0 W^2 - \alpha + \frac{\varkappa}{2}(u_1^2 + u_2^2) + g\right]u_1 + A_1 \equiv L(u_1, u_2) \\
\rho_0 \nu W \frac{du_2}{d\xi} &= \left[\rho_0 W^2 - \alpha + \frac{\varkappa}{2}(u_1^2 + u_2^2) - g\right]u_2 + A_2 \equiv M(u_1, u_2) \\
A_1 &= -U_1(\rho_0 W^2 - \alpha + {}^1/_2\varkappa R^2 + g) \\
A_2 &= -U_2(\rho_0 W^2 - \alpha + {}^1/_2\varkappa R^2 - g), \quad R^2 = U_1^2 + U_2^2
\end{aligned} \tag{2.2}$$

For the problem of structure to have a solution, integral curves connecting the point $A(U_1, U_2)$ representing the state in front of the discontinuity $(\xi = -\infty)$ with the point u_1, u_2 depicting the state behind the shock $(\xi = +\infty)$ in which $(du_\alpha/d\xi)_\infty = 0$ should exist for system (2.2). Consequently, we first find the stationary points of the system (2.1) where $du_\alpha/d\xi = 0$. One of them corresponds to the state in front of the shock, the rest correspond to a possible state behind the shock for a given value of W, since they correspond to the same momentum fluxes in the direction of the x_1 and x_2 axes. On the lines where one of the equalities $M(u_1, u_2) = 0$, $L(u_1, u_2) = 0$ (isoclinic) is satisfied, the tangents to the integral curves are parallel to the u_1 and u_2 axes, respectively. Investigation of the shape of these curves shows that for a fixed state in front of the discontinuity, the isoclinic $du_\alpha/d\xi = 0$ can consist of one or two branches intersecting the u_α axis at a right angle and symmetrical to it, depending on the magnitude of the shock velocity W.

Represented in Fig.1 is one of the possible locations of the isoclinic $du_\alpha/d\xi = 0$. Depending on the quantity W, the oval and the unclosed branch can change places in the expressions for M and L. There may generally be no oval. Intersection of the lines $L = 0$ and $M = 0$ yields the location of the singular points of (2.3), i.e., the stationary points of system (2.2).

For a given state on one side of the discontinuity $u_1 = U_1$, $u_2 = U_2$ and arbitrary W all the possible states on the other side of the discontinuity had been found earlier /9/ in the form of a shock adiabatic that is displayed by the curve in the u_1u_2 plane (Fig.2a). The dependence of W on points of the adiabatic (Fig.2b) indicates the correspondence between the value of W and the state behind the shock (the parameter φ in Fig.2b is a certain coordinate that changes monotonically along the shock adiabatic). The behaviour of the shock depends very much on the sign of the elastic constant $\varkappa$ of the medium. To be specific, $\varkappa > 0$ is taken in all the later discussions. The quantities c_α^- $(\alpha = 1, 2)$ noted on the W axis in Fig.2b are the characteristic velocities of the slow and fast quasitransverse waves /12/ in the sate in front of the shock, i.e., for $u_\alpha = U_\alpha$

$$\rho_0 c_{1,2}^2 = \alpha - {}^1/_2\varkappa\{u_1^2 + u_2^2 \pm [(u_1^2 - u_2^2 + g/\varkappa)^2 + 4u_1^2u_2^2]^{1/2}\}$$

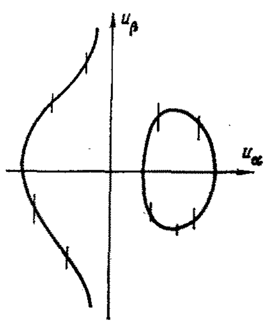

Fig.1

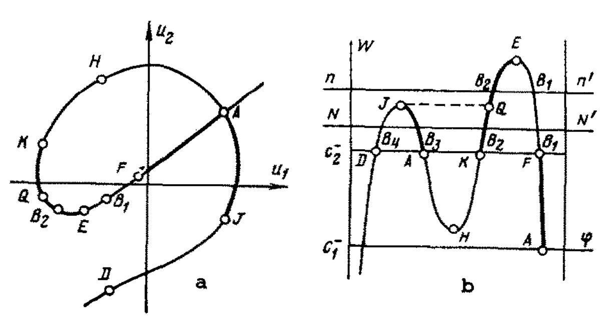

Fig.2

The heavy lines in Fig.2 mark those sections of the adiabatic and the velocity graph that correspond to the evolutionary shock. Only for these values of the shock velocity W must the discontinuity structure be investigated, since shocks are known not to be able to exist for non-evolutionary sections.

The discontinuities for which $W_J < W < W_E$ (Fig.2b) are of particular interest. The evolutionary shocks satisfying this condition are fast and the state behind them is represented by points of the segment QE of the shock adiabatic (Figs.2a and b). A condition for the existence of the segment QE is the inequality $W_J < W_E$, which is satisfied if the quantities $U_\alpha/(g/\varkappa)^{1/2}$ emerge from a certain bounded range of values /9/. As is shown in /6/, the self-similar problem can have two solutions under these same conditions, where one of them contains a shock corresponding to one of the points of the segment QE. Moreover, this solution can contain a slow shock. The second solution contains discontinuities corresponding to the points J and K of the shock adiabatic and can also contain a slow shock. All the discontinuities listed, except those that correspond to points of the segment QE also enter into the other solutions when these solutions are unique, and consquently, as has already been mentioned above, their existence can hardly be subject to doubt. The absence of a stationary structure for these waves should have denoted that a non-stationary structure exists.

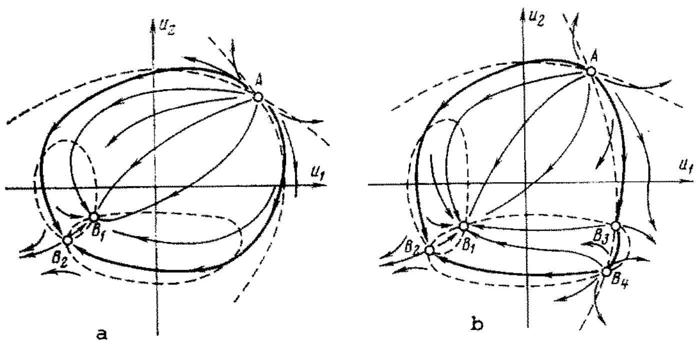

Fig.3

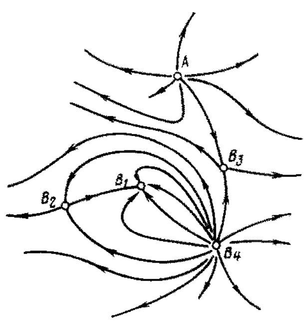

Fig.4

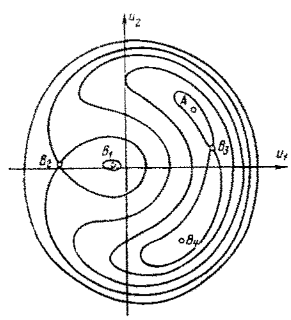

Fig.5

Knowledge of the shock adiabatic and graphs for W assists in establishing the location of the singular points of (2.3). Selecting a certain value $W = \text{const}$, we thereby draw lines $NN', nn', \ldots$ in Fig.2b that all intersect the velocity graphs at the points B_i. There can be two such points (B_1, B_2 on the line nn'), four (B_1, B_2, B_3, B_4 on the line NN') or none. States behind the shock denoted by the same letters B_i correspond to the points mentioned on the shock adiabatic (Fig.2a). Together with the point A these points will indeed be stationary points of system (2.2) for a given value of W. In order to know which shock transitions from the initial state are possible, it is necessary to indicate those among the singular points B_i (if they exist) at which integral curves departing from the point A will arrive as ξ grows. To do this it is necessary to clarify the kinds of singular points A, B_i and to investigate the field of directions $\{L, M\}$.

The curves $L = 0$, $M = 0$ which divide the u_1u_2 plane into domains with different slopes of the vector $\{L, M\}$ for the case when $W_J < W < W_E$ which corresponds to the location of the points B_1 and B_2 in Figs.2a and b, are displayed by dashes in Fig.3a. The qualitative pattern of the integral curves is shown in Fig.3a by the solid lines. The point A is a node with emerging integral curves, point B_1 is a node with entering integral curves, while point B_2

belonging to the segment QE is a saddle. The transition $A \to B_1$ corresponds to a non-evolutionary shock while the transition $A \to B_2$ is a fast shock. It is easy to conceive that two integral curves going from A to B_2 exist. Using the idea of /13, 14/, we will show that these solutions will exist even for any positive-definite matrix $\nu_{\alpha\beta}$ since the type of singular points does not change when going from $\nu\delta_{\alpha\beta}$ to $\nu_{\alpha\beta}$ and two rays of the separatrix of the point B_2 going towards decreasing q should arrive at the point A, since there is a single minimum of the function q at this point and q increases with distance from the origin.

In the case $c_2^- < W < W_J$, the singular points B_1, B_2, B_3, B_4 in Fig.2b lie on the line NN'. Their location in the u_1u_2 plane is shown in Fig.3b. The discontinuities corresponding to the transitions $A \to B_2$ and $A \to B_3$ will be evolutionary. Now there are to minima of the function q in the u_1u_2 plane, at the points A and B_4, and two saddle points B_2 and B_3. It always turns out that $q(B_3) < q(B_2)$. Here (and for any matrix $\nu_{\alpha\beta}$) the existence of an integral curve going from the point A to the point B_3 can be ensured. Indeed, let us examine the line $q(u_1, u_2) = C$. For values of C slightly exceeding $q(A)$ this line is closed and encloses the point A. An integral curve leaving A passes through each point of it. As C increases, the surface $q = C$ will pass through the point B_3, which ensures the existence of the solution under consideration.

The existence of an integral curve going from point A to point B_2 depends on the behaviour of the separatrix emerging from the point B_3 (saddle point). The separatrix mentioned separates the integral curves emerging from the nodes A and B_4. If it enters the point B_1, as is shown in Fig.3b, then the points A and B_2 are connected by an integral curve. Otherwise (Fig.4), the stationary shock structure corresponding to the points B_2 (these are fast waves belonging to the segment KQ of the shock adiabatic) does not exist.

The behaviour of the separatrix is determined by the matrix $\nu_{\alpha\beta}$. Since $q(B_2) > q(B_3) > q(A)$, it is always possible to find $\nu_{\alpha\beta}$ such that the integral curve from A to B_2 will not exist, and for which this integral curve will exist. Naturally the scalar, i.e., isotropic, viscosity and the case of slightly anisotropic viscosity are of greatest interest. The structure of all evolutionary shocks exists for these cases.

Patterns of the level lines $q(u_1, u_2) = \text{const}$ were computed on a computer for all possible values of the parameters. It is seen from a typical pattern (Fig.5) that for the separatrix from the point B_3 not to fall at the point B_1 it should make a very small angle with the lines $q = \text{const}$.

The case $W_H < W < c_2^-$ differs from that considered above in that the points A and B_3 in Figs.3b and 4 and 5 change places, where computations on a computer confirm that the structure of a slow shock wave exists for a scalar, or slightly different from a scalar, viscosity. In the case $c_1^- < W < W_H$ the existence of a slow shock structure is proved rigorously for any anisotropic viscosity. A stationary structure for fast shock waves corresponding to the segment AJ in Figs.2a and b exists in exactly the same way.

Therefore, for isotropic viscosity as in (2.1) and in the case when the viscosity is slightly anisotropic, the existence of a stationary structure is confirmed for all quasi-transverse shock waves by computer calculations. For shock waves "responsible" for the ambiguity of the solution of a selfsimilar problem, for slow shock waves for $c_1^- < W < W_H$ and fast shock waves corresponding to the section AJ of the shock adiabatic, the existence of a stationary structure is proved rigorously for any anisotropic viscosity.

An analogous investigation for media with $\varkappa < 0$ shows that at least for slow shock waves occurring in the composition of one of the solutions of the selfsimilar problem in the non-single-valued domain and not taking part in any solutions of this problem for other domains, a stationary structure exists.

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A METHOD OF INVESTIGATING WEAKLY NON-LINEAR INTERACTION BETWEEN ONE-DIMENSIONAL WAVES*

A.V. KRYLOV

A method of constructing asymptotic approximations of wide classes of solutions of weakly non-linear systems is proposed based on the averaging scheme developed in /1-3/.**(**See also: Krylov A.V. and Shtaras A.L. Internal averaging of multidimensional weakly non-linear systems along characteristics, Dep. in LitNIINTI, 10.11.86, No.1750, 1986). The method enables one to obtain the conditions for the asymptotic decay of systems described by the Burgers, Korteweg-de Vries and similar scalar equations, and also enables one to investigate problems in which this decay does not occur. As an example we investigate the propagation of perturbations in an elastic non-uniform tube. The interaction between two waves is considered and the conditions for resonance are obtained.

1. Non-linear wave phenomena are usually studied using simplifying assumptions of a heuristic form. Hence, a theoretical justification is necessary as well as an investigating of the limit of suitability of the solutions obtained.

Suppose the solution of the quasilinear system

$$U_t + A(U) U_x = 0, \quad U = (u_1, \ldots, u_n), \quad A(U) = \| a_{ij}(u_1, \ldots, u_n) \| \tag{1.1}$$

is close $(0 < \varepsilon \ll 1)$ to a certain state of equilibrium $(U_0 \equiv \text{const})$

$$U = U_0 + \varepsilon U_1(t, x, \varepsilon) \tag{1.2}$$

We assume that the constants (const) are everywhere independent of ε; the subscripts i and j take the values $1, 2, \ldots, n$.

If problem (1.1), (1.2) is hyperbolic (/4/, p.23), then by making the replacement $V = RU_1$, $R = \| r_{ij} \|$, $\det R \neq 0$ it can be reduced to the form

$$V_t + \Lambda V_x = -\varepsilon R A_1 [R^{-1} V] R^{-1} V_x + o(\varepsilon) \tag{1.3}$$

$$\Lambda \equiv \operatorname{diag}\{\lambda_1, \ldots, \lambda_n\} = RA(U_0) R^{-1}$$

$$A_1[U_1] = \frac{dA(U_0)}{dU} U_1 \equiv \left\| \sum_{k=1}^{n} \left[\frac{\partial}{\partial u_k} a_{ij}(U_0) \right] u_{1k} \right\|$$

The initial condition

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